Point-Nine Factor for Near Surface Flow Measurements

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Introduction

LaserFlow, a Non-Contact Open-Channel Laser-Doppler flowmeter, can focus at the center of a flow channel below but near the surface of the water, where the maximum velocity stream typically occurs. In Area-Velocity flow measurements, the Volumetric Flow, Q, of the water stream is determined from the product of the wetted cross-sectional area and the average velocity of the flow profile, according to the Continuity equation, i.e. $Q = A_{wet} * U_{avg}$. It has been known that for typical open-channel turbulent flow[5], the average velocity of the flow profile is very well approximated by scaling the maximum velocity of the profile by 90%, i.e. $U_{avg} = 0.90 * U_{max}$. In this poster, technical literature references and empirical evidence are cited in support of the Point-Nine Factor.

Technical Literature

The US Bureau of Reclamation in their Water Measurement Manual teaches the Subsurface method as early as the 1967 edition[1]. The manual suggests measuring the flow velocity near the subsurface and scaling by a coefficient between 0.85 and 0.95. The text does caution on the accuracy of this method. Grant and Dawson, Teledyne ISCO Open Channel Flow Measurement Handbook[3] quotes the Subsurface method on p. 142. The ISO 748-2007 Standard[2], under "Surface one-point method" (7.1.5.3.7.1), recommends the coefficient value between 0.88 to 0.90 "when bed is smooth." Quoting from the abstract of the US Patent 5,198,989[4], "...determining average velocity to be approximately 90% of the peak velocity." Chao-Lin Chiu in TABLE 1 of his 1988 paper[5] quotes flow data from an earlier publication (Blaney 1937) on the ratio between average velocity against maximum velocity, U_{avg}/U_{max}, for twelve canals. The values, in Column 8 of his Table on page 748, range from 0.92 to 0.79 with a median value at 0.875.

[1] Water Measurement Manual, 3rd ed. **2001,** US Department of Interior, Bureau of Reclamation, p. 10–16.

[2] ISO 748-2007 Standard, 7.1.5.3.7 Surface one-point method.

[3] Grant and Dawson, Teledyne ISCO *Open Channel Flow Measurement Handbook* 8th ed. **2017,** p. 142.

[4] US Patent 5,198,989

[5] Chao-Lin Chiu, *J. Hydraul. Eng.,* 114.7, **1988,** pp. 738–756.

LaserFlow Data

A LaserFlow sensor was used to collect flow data at Alden Research Laboratory in Holden, MA between 11 and 21 July 2016. This flowmeter was tested on the Alden Hooper Facility Line 3 with a 36" round pipe. The flow measurements are compared against the Alden Lab's 36" Master Venturi and 8" Venturi flow standards. The LaserFlow averaged 3 points in an L-shape near the subsurface of the flow before scaling with the "B" coefficient at 0.90. The plot below contains 40 averaged data points and shows an average absolute % error near 4.3%. Please see Figure 1.

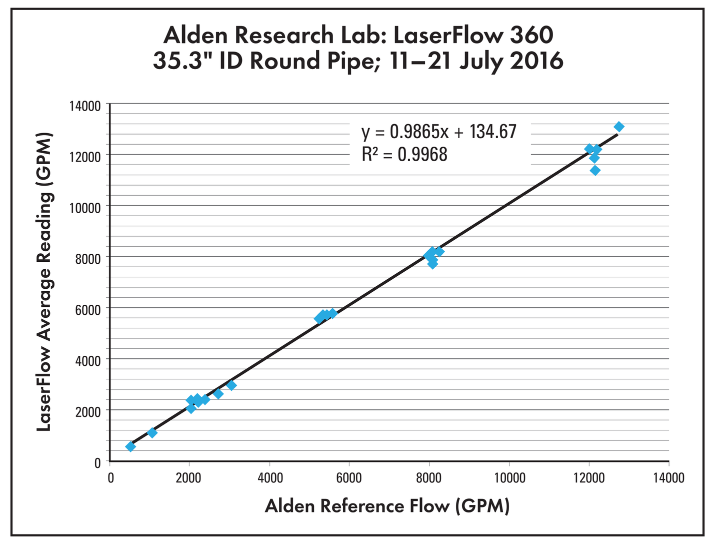


Figure 1

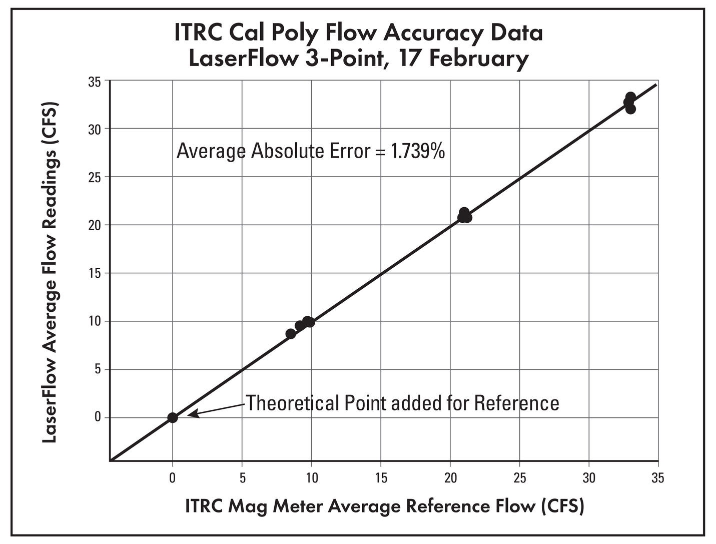


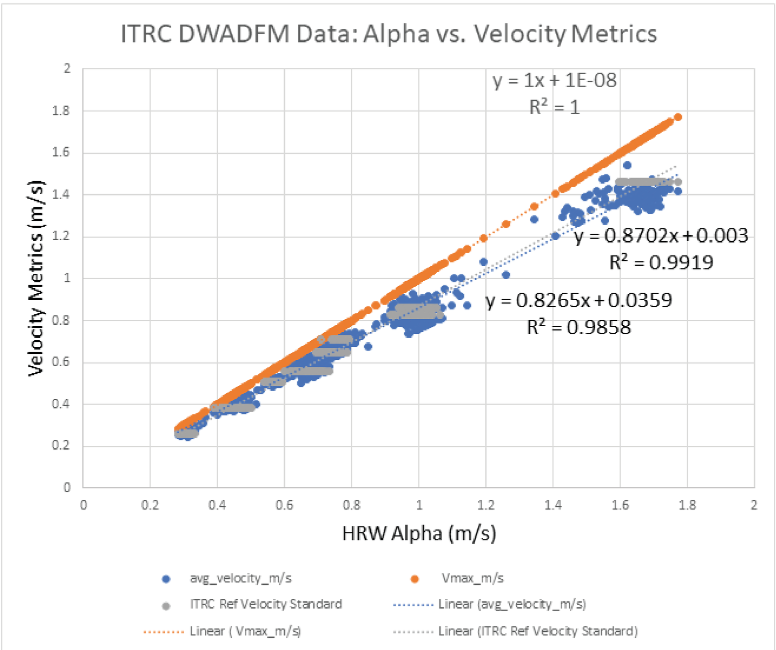
Figure 2

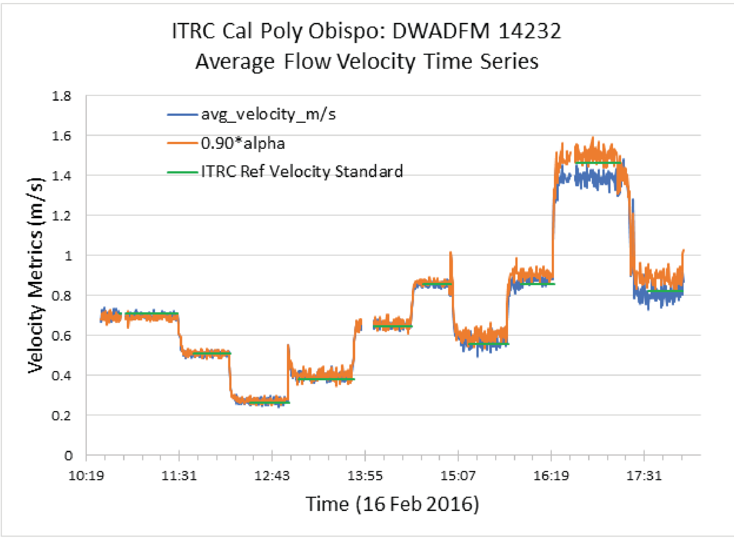
A LaserFlow sensor was also used to collect flow data at the Irrigation Training and Research Center (ITRC) on the California Polytechnic State University campus in San Luis Obispo, CA between 16 to 18 February 2016. The flowmeter was tested on the 48-inch wide square-channel line using a 30-inch McCrometer Ultramag as the flow standard. Their weigh tank primary standard was also used to verify a few data points. The LaserFlow averaged 3 points in an L-shape near the subsurface of the flow before scaling with the "B" coefficient set at 0.90. The plot below contains 10 averaged data points plus a theoretical (0,0) point. During the 17 February 2016 test, it showed an average absolute % error near 1.7%. Please see Figure 2.

Acoustic Data

While at the ITRC, the flow accuracy of the Teledyne ISCO DWADFM acoustic pulsed Doppler flowmeter was also tested on the 48-inch wide square-channel line. In this meter, the flow velocities for 9-point locations across the wetted cross-sectional area of the flow are measured by time-of-flight acoustic Doppler sonar. Knowing the wetted cross-sectional geometry, the velocity values at each of the 9-point locations is fitted to a H.R. Wallingford (HRW) profile model [US Patent 7,267,013]. The HRW model is a combination of powers of sine and cosine functions with coefficient parameters, which conforms to the flow velocity profile over the 2-D wetted cross-sectional area of the flow stream. After the regression fit, the HRW is integrated over the wetted area to determine the Flow, Q, value. Dividing the Flow, Q, by the wetted area, A_{wet}, gives the average flow velocity, U_{avg}. One coefficient parameter in the HRW, Alpha, can represent the maximum velocity, U_{max}, of the flow profile. Please see Appendix. U_{max} is also determined numerically by searching for extrema on the HRW model surface after the regression fit.

Using DWADFM data collected at ITRC on 16 February 2016, the real-time average velocity readings, U_{avg}, are plotted along with the HRW "0.90*alpha" values and for reference, the average velocity readings determined from the ITRC McCrometer Ultramag standard. A version of the HRW model is used that is the same as the Appendix. For square channels, Kappa is set to 0.56 and Delta is set to 0.05. Furthermore, the flow is assumed to be symmetric, so Gamma is set to zero. Secondary flows are ignored, so Eta is set to zero. Alpha and Beta fit the simplified HRW model to the 9-point DWADFM Doppler readings. After the fit, U_{max} is determined by numerically searching over the model. U_{max} and Alpha values are plotted to verify the derivation in the Appendix. U_{avg}, Alpha and the ITRC Reference values are also plotted together. The least-squares slope for the Alpha vs. U_{avg} linear plot is in the lower 0.8 range, but Alpha vs. the ITRC reference standard values is in the upper 0.8 range. Please see plots below.





ISO 748 Reference Data

At the USGS HIF in Stennis Space Center, MS, the flow reference on their Large Tilting Flume is based on the ISO 748 standard and 27 velocity readings from a Flow Tracker acoustic velocimeter. For flow levels at or above 2 feet, the ISO 748 Three-Point (80/60/20) method is used for each of nine equally-spaced stations across the 5-foot wide square channel. The table below shows results for U_{avg}, U_{max} and their ratios under both the ISO 748 and HRW models.

Under the ISO 748 model, U_{avg} is calculated as the average velocity for each station. For each station, the velocity is calculated according to the ISO 748-2007 Three-Point method (7.1.5.3.4, eq. 5). U_{max} is simply the maximum value from all 27 readings regardless of position. Unfortunately, the readings that are closest to the surface are only 80% of the water level. The maximum velocity flow stream for a square channel is expected to be near the 90% water level region, so this estimate for U_{max} is expected to be slightly too low. The ratio values, near 0.95, in the table are probably slightly too high as a result.

Under the HRW model, five of its parameters (Alpha, Beta, Gamma, Delta and Eta) are allowed to freely fit the 27 velocity readings by Marquardt non-linear least-squares regression. The parameter, Kappa, is set constant to 0.56, assuming that the maximum flow velocity stream is near the 90% water level, typical of square channel flow. After regression, the HRW model is integrated over the wetted cross-sectional area of the flow using the Simpson One-Third Rule to calculate the flow Q. U_{avg} is calculated by dividing Q by the wetted area, A_{wet}, i.e. Water Level * 5.00 feet wide. U_{max} is determined by a numerical search of the maximum on the HRW model over the wetted area. The ratio values in the table are now closer to 0.90.

Settings		ISO 748 3-Point: 80/60/20			HRW Model (κ=0.56, n=27)		
Level (inches)	Flow (CFS)	U_{max} (ft/s)	U_{avg} (ft/s)	U_{avg}/U_{max}	U_{max} (ft/s)	U_{avg} (ft/s)	U_{avg}/U_{max}
32	10	0.8543	0.7982	0.9343	0.8556	0.7663	0.8955
32	17	1.3648	1.2758	0.9348	1.3710	1.2328	0.8992
32	39	3.1463	3.0036	0.9546	3.1990	2.9051	0.9081
26	14	1.3127	1.2511	0.9531	1.3136	1.2039	0.9164
25	20	2.0220	1.9055	0.9424	2.0415	1.8398	0.9012
24	30	3.0105	2.8718	0.9539	3.0290	2.7709	0.9148
24	39	4.0167	3.8722	0.9640	4.1022	3.7274	0.9086

Conclusions

A pictorial proof of the 0.9-Factor can be seen in Figure 3. During laminar flow, the velocity profile is parabolic and the ratio, U_{avg}/U_{max} is nearly 0.67. Integrating this sagittal section for a full round pipe, the ratio is 0.5. As the flow becomes turbulent, eddy currents form and flatten the profile, approaching Prandtl's one-seventh power law. Under turbulent flow, this ratio approaches 0.88 to 0.90. The 0.9-Factor in the LaserFlow "B" coefficient provides accurate flow measurement for most applications. With additional information, the factor's value can be better optimized.

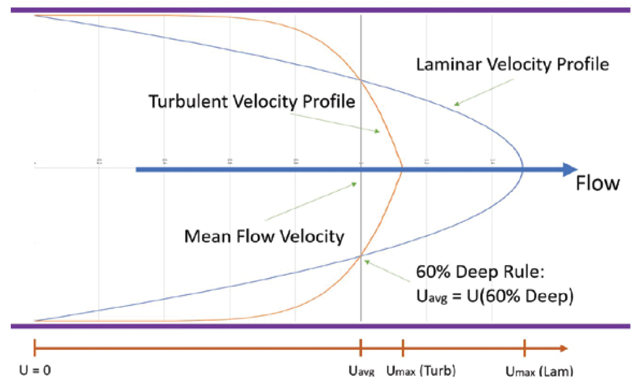


Figure 3: Flow Velocity Profiles

Appendix

Physical Meaning of the HRW Alpha – Consider a simplified version of the HRW, assuming symmetry (no skewness) with no secondary flows, i.e. $\gamma=\eta=0$:

$$U(\varphi,\theta) = \alpha \sin^{\beta}(\kappa\varphi) \cos^{\delta}(\theta)$$

A velocity maximum (extrema) occurs when $\partial U/\partial\varphi = 0$ and $\partial U/\partial\theta = 0$. Partially differentiating the HRW and solving for the above conditions, one obtains:

$$\cos(\kappa\varphi) = 0, \text{ when, } \varphi=\pi/(2\kappa), \text{ and } \sin(\theta) = 0, \text{ when, } \theta=0.$$

So:

$$U_{max} = U(\pi/(2\kappa),0) = \alpha \sin^{\beta}(\cancel{\kappa}\ \pi/(2\cancel{\kappa}))\cos^{\delta}(0) = \alpha.$$